High-performance algorithms for double difference data processing

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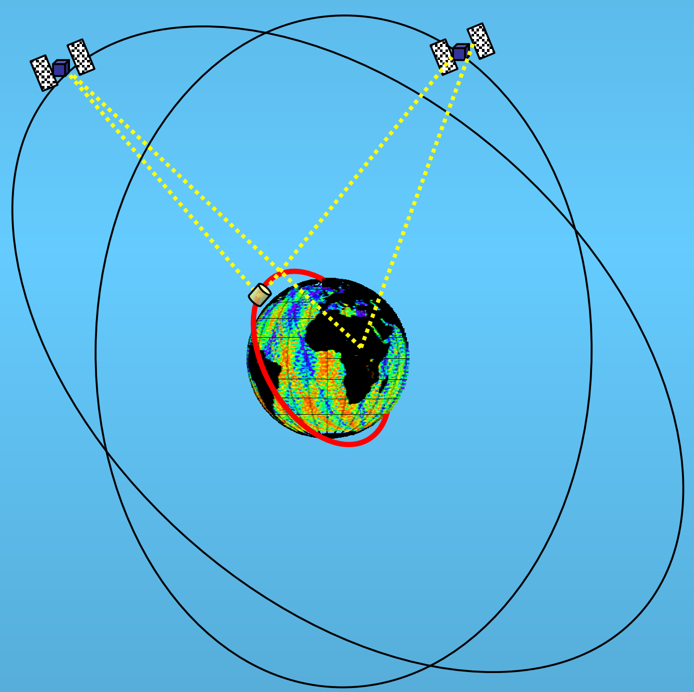
Double difference properties

- Traditionally, “redundant” DD combinations are avoided:
  - Linear dependence of normal matrix information
  - Imbalances in station weighting
  - Redundant workload
- In fully correlated estimation of orbits, clocks and ambiguities “redundant” DD combinations are *not* redundant
  - DD residuals are all independent
  - Orbit, ambiguity information is amplified, clocks are not



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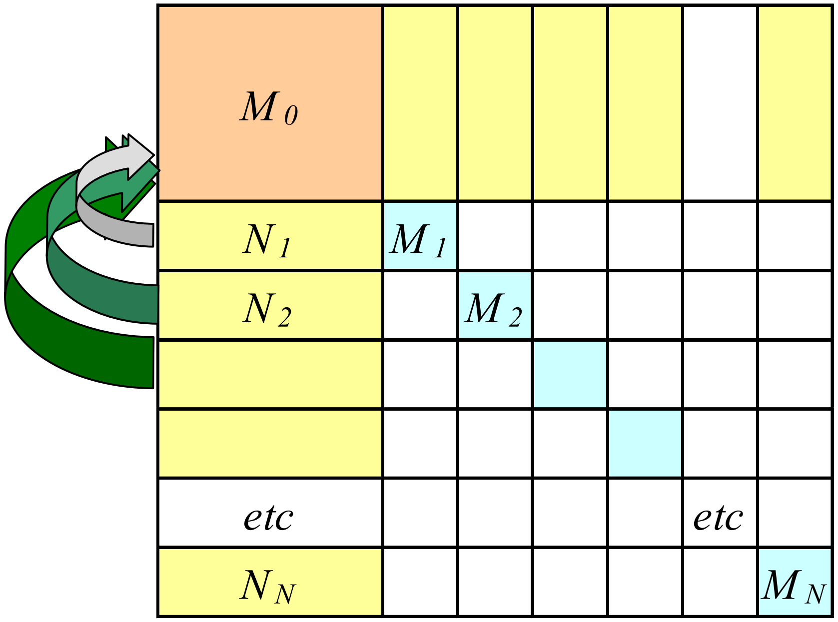
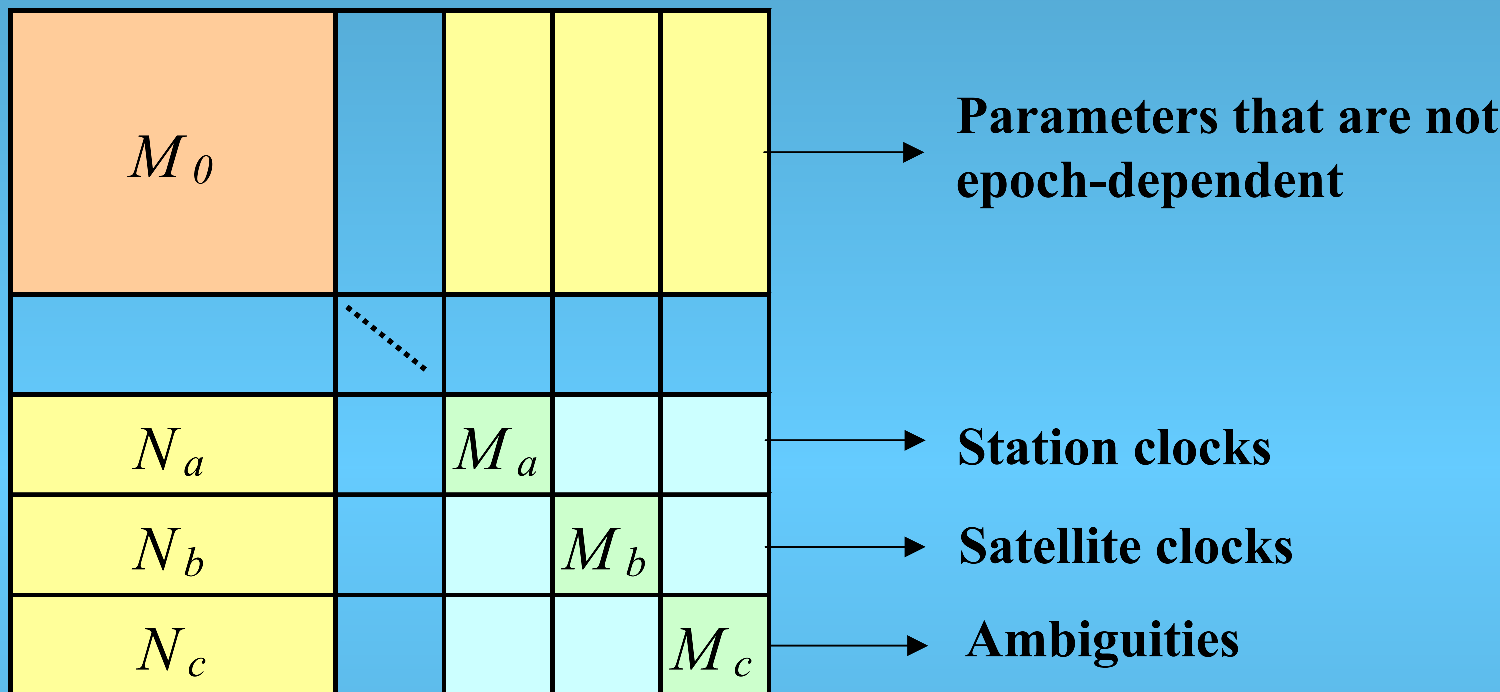
Normal matrix partitioning (1)

- Normal equation looks like $M\vec{x} = \vec{y} \;\Rightarrow\; \vec{x} = M^{-1}\vec{y}$
- Epoch-dependent parameters:
  - Station and satellite clocks (fixing one per epoch)
  - Phase ambiguities (constrained to value from previous epoch)
- Main partition M_0 is for all other parameters (… may have further partitions)



1. Matrix partition for orbit parameters
2. Matrix partitions for epoch parameters
3. Partitions that correlate 1. and 2.

Normal matrix partitioning (2)



- Separate partitions M, N exist for every epoch
- M_a, M_b, M_c are diagonal matrices
- No correlations between epochs other than ambiguity constraints

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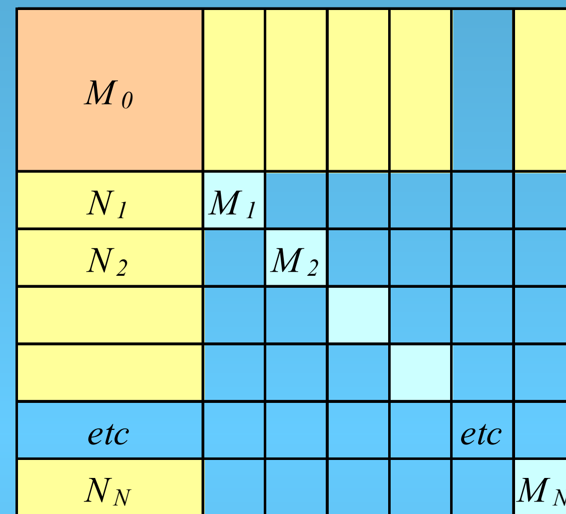
Normal matrix partitioning (3)

$$M\vec{x} = \vec{y} \;\Rightarrow\; \vec{x} = M^{-1}\vec{y}$$

M, x and y are partitioned for epoch parameters



$$\begin{cases} M_0\vec{x}_0 & +N_1^t\vec{x}_1 & +\ldots & +N_n^t\vec{x}_n = \vec{y}_0 \\ N_1\vec{x}_0 & +M_1\vec{x}_1 & & = \vec{y}_1 \\ \vdots & \vdots & & \vdots \\ N_N\vec{x}_0 & +M_n\vec{x}_n & & = \vec{y}_n \end{cases}$$

Each x_i can be expressed in x_0: $\vec{x}_i = M_i^{-1}\left(\vec{y}_i - N_i\vec{x}_0\right)$

$$\Rightarrow \left(M_0 - \sum_{i=1}^{n} N_i^t M_i^{-1} N_i\right)\vec{x}_0 = \vec{y}_0 - \sum_{i=1}^{n} N_i^t M_i^{-1}\vec{y}_i$$

From this system <u>only</u> the vector x_0 is solved explicitly – not the many x_i

Normal matrix partitioning (4)

- Once that vector x_0 is available, the *a posteriori* orbit can be propagated
- New y_i are computed at every epoch, and no longer contain errors due to the *a priori* x_0
- Epoch vectors x_i can now be solved from the local system
- Partitions N_i are only needed once, and do not have to be stored or recomputed
- Number of epochs can now be very large: high-rate data

$$\vec{x}_i = M_i^{-1}\left(\vec{y}_i - N_i\vec{x}_0\right)$$

$$\Downarrow$$

$$\vec{x}_0 = F\left(\vec{y}_0, M_0, N_i^t M_i^{-1}\vec{y}_i\right)$$

$$\Downarrow$$

New orbit

$$\Downarrow$$

New vectors y_i

$$\Downarrow$$

$$\vec{x}_i = M_i^{-1}\vec{y}_i$$

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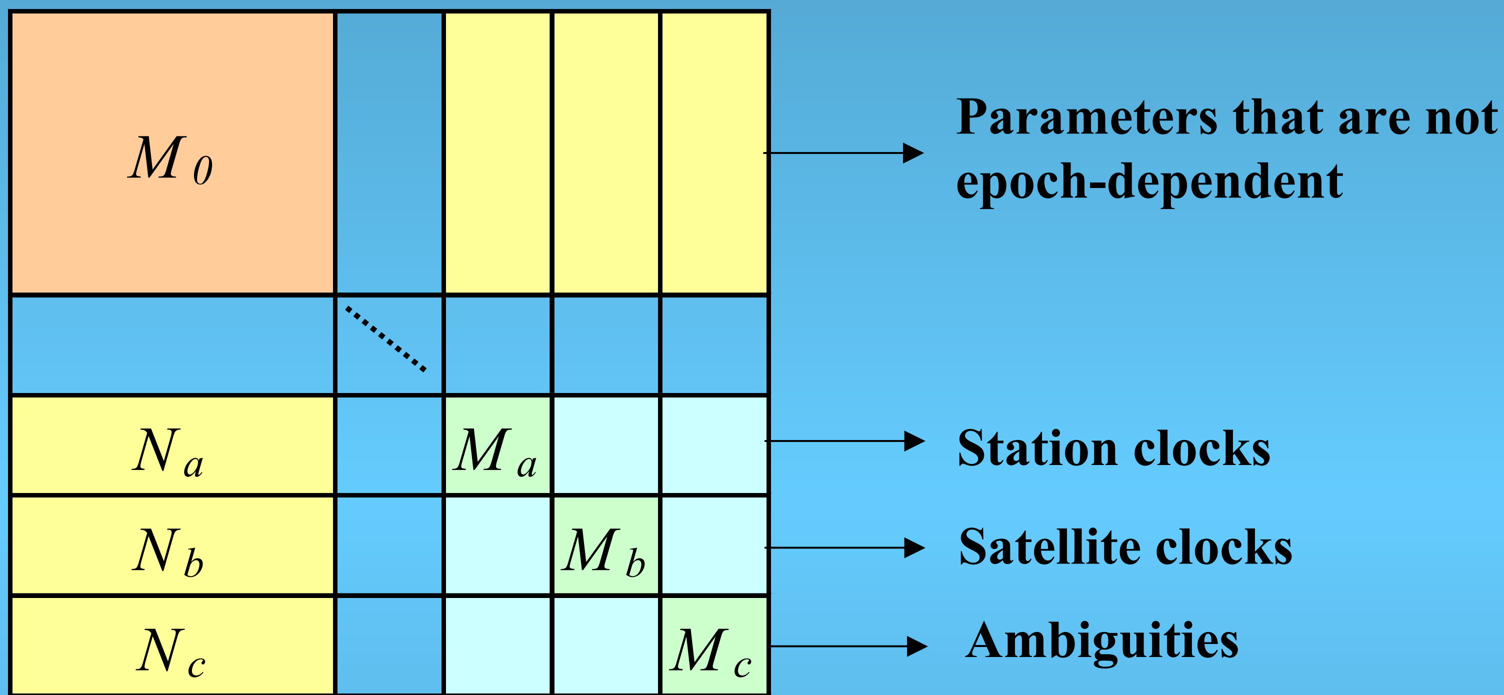
Double difference information (1)



- Undifferenced information enters in all matrix partitions
- Code DD information enters in M_0 only
- Phase DD information enters in M_0, N_c, M_c

Double difference information (2)

- "Redundant"combinations are DD observations that share one or more undifferenced observations:

  E = (A – B) – (C – D)

  F = (A – B) – (C – G)

  H = (D – B) – (I – G) etc.

- Each DD observation affects a different combination of matrix rows in the fully correlated partitioning
- "Redundant" combinations are *not* redundant, but help to separate orbits, clocks and ambiguities by amplifying different parts of the normal matrix system in different ways

Handling of DD observations (1)

DD observation equation: $E = (A - B) - (C - D)$

$$R_E = (R_A - R_B) - (R_C - R_D)$$

Normal matrix contribution:

$$E^tE = E^tR_E$$

$$= A^tA - 2A^tB - 2A^tC + 2A^tD + B^tB - 2B^tC$$

$$- 2\,B^tD + C^tC + 2\,C^tD + D^tD$$

- The ten right-hand side terms occur in many DD combinations
- Contributions to M_0 are always the same even for phase & code!
- Contributions N_c, M_c (ambiguities) are different for all DD
- Instead of producing the costly products A^tA, A^tB, etc. many times, it is sufficient to count how often they are needed in M_0
- Products are generated just once, and multiplied with their count

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Handling of DD observations (2)

- For any four undifferenced observations:
  - There are four contributions A^tA , B^tB , B^tB , B^tB
  - There are *at most* six mixed terms A^tB, A^tC, etc
- Counting of DD combinations represents a negligible workload compared to the actual matrix accumulation
- Workload for handling *all possible* double differences can never exceed 10 / 4 times the workload for handling the undifferenced observations. In practice, spherical Earth prevents ~60% of the combinations.
- Workload for processing DD data never exceeds about twice the workload for undifferenced processing, regardless of the amount of DD combinations that are included!

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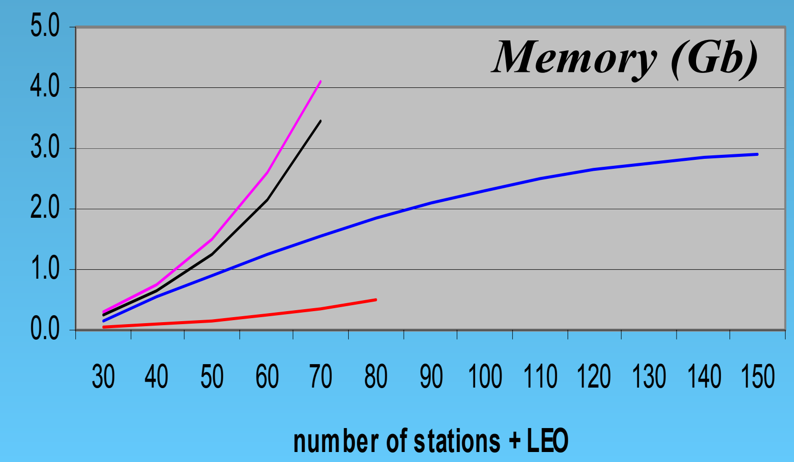
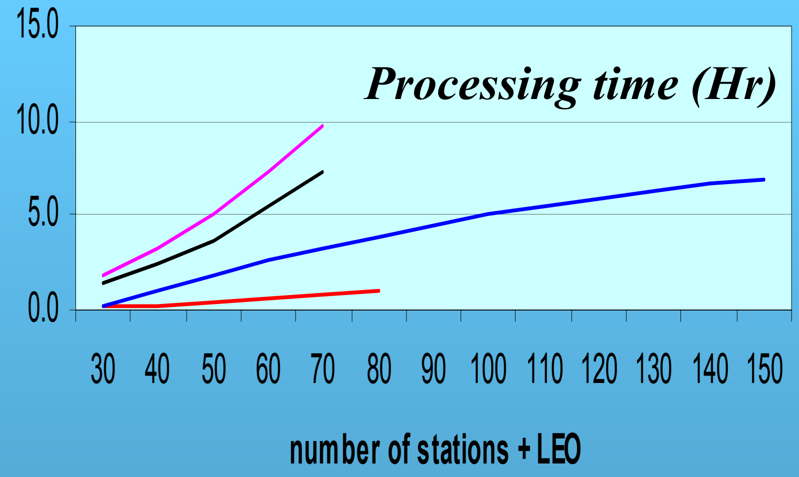
Workload for DD handling

- The number of geometrically possible DD combinations is a quadratic function of the number of undifferenced observations (… stations)
- The workload for DD handling is linear in the number of ***un***differenced observations (~factor 2)
- The workload for DD handling is only a square root function of the actual amount of DD combinations!





Example of DD performance

Test platform: **SUN fire V480 server**	Typical POD run of ESOC IGS Analysis Centre				
		ROBOD prototype			
		IGS-like process		*Large scale process*	
	without DD	*without DD*	*with DD*	*without DD*	*with DD*
GNSS satellites	*28*	*28*	*28*	*38*	*38*
LEO satellites	*0*	*0*	*0*	*2*	*2*
IGS ground stations	*52*	*59*	*59*	*150*	*150*
arc length (hrs)	*48*	*24*	*24*	*24*	*24*
epoch interval (s)	*300*	*150*	*150*	*15*	*15*
nr of epochs	***576***	***576***	***576***	***5,760***	***5,760***
clocks + phase ambiguity parameters	*40,602*	*54,720*	*138,832*	*91,332*	*485,228*
other estimated parameters	*524*	*1,119*	*1,119*	*2,534*	*2,534*
total parameters	***41,126***	***55,839***	***139,951***	***93,866***	***487,762***
undifferenced observations	*441,728*	*563,658*	*563,658*	*7,877,928*	*7,877,928*
double difference observations	*0*	*0*	*11,409,520*	*0*	*213,573,080*
total observations	***441,728***	***563,658***	***11,973,178***	*7,877,928*	*221,451,008*
Ratio observations / parameters	***11***	***10***	***86***	***84***	***454***
Memory (Mb)	*583*	*141*	*249*	*824*	*2,488*
Processing time (hrs:min:sec)	*03:40:27*	*00:05:21*	*01:04:03*	*01:27:47*	*14:24:30*

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Conclusions

- In fully correlated estimation of orbits, clocks and ambiguities, “redundant” DD combinations still contain useful information
- Efficient handling of epoch-dependent matrix partitions allows for high-rate data (… LEO)
- Efficient handling of large amounts of DD observations is possible if properties of linear combinations are exploited
- Workload for DD processing is less than proportional to the actual number of DD observations
- Number of considered DD combinations is virtually unlimited, all geometrically possible DD are included
- Prototype implementation illustrates high performance, precision improvements are still needed

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