# Kinematic Orbit Determination for Low Earth Orbiters (LEOs)

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**Abstract.** Kinematic point positioning of a Low Earth Orbiter (LEO) using GPS data is one possibility to get precise orbit information. This approach is followed at the Astronomical Institute of the University of Bern (AIUB) as an alternative to the dynamical orbit determination. Kinematic point positioning allows to recover the trajectory of the LEO without making use of any a priori gravity field information. This may be very useful for gravity field recovery, in particular in view of present and upcoming satellite missions like CHAMP, GRACE and GOCE which all have an accelerometer on board.

The emphasis of this paper is to study the effect of different data screening options on the quality of the kinematic orbit for a LEO. The impact of observations at low elevations in conjunction with elevation-dependent weighting is investigated. The tests are carried out using data from SAC-C and CHAMP. Comparison with dynamic orbits of the satellites indicate that a kinematic LEO orbit at the decimeter accuracy level is feasible provided good code and phase GPS data is available.

**Keywords.** Low Earth Orbiter, kinematic orbit determination, preprocessing

## 1. Introduction

The AIUB has a well documented experience in processing data of GPS receivers on the Earth's surface. Since 1992 it is the home of the Center for Orbit Determination in Europe (CODE) as one of the IGS (International GPS Service) analysis centers. Two years ago we started to process GPS data from spaceborne receivers like the one on GPS/MET or TOPEX/POSEIDON. To this purpose a procedure for kinematic orbit determination using GPS code and phase data was developed.

The approach for the extraction of kinematic satellite trajectories currently implemented at AIUB is based on an epoch-wise processing of the code observations combined with an epoch-difference processing of the phase observations. The procedure is efficient because no ambiguity parameters need to be solved for. A good quality of the data is important for this approach. Because GPS flight receivers are currently tracking only up to eight satellites simultaneously, an elaborated procedure for data screening is needed. In order to use a maximum number of observations the elevation cut-off angle may be set to a low value. On the other hand the observations at low elevations may be affected by ionospheric refraction and multipath effects.

## 2. Kinematic Determination of Orbits

As for receivers on the ground a number of different approaches allow to compute point positions for a flying GPS receiver using its code and phase observations. Common to most approaches is the introduction of GPS orbits and clock corrections as fixed. Data may be processed on the zero-difference level or on the double-difference level after forming baselines from the LEO to different ground stations (see Svehla *et al.*, 2001). Processing of phase observations usually requires the estimation of ambiguity parameters in which case the kinematic positions based on phase only may be an interesting option. The result of all procedures is a satellite trajectory (usually called kinematic orbit), which is independent of any a priori force field information.

The approach currently followed at the AIUB avoids the setting up of ambiguity parameters by forming differences of the phase observations from one epoch to the next. The algorithm is described in detail in Bock *et al.* (2000). GPS data is processed at the zero-difference level using the ionosphere-free linear combination. Positions derived from code and position differences from phase epoch-differences are combined in order to generate the kinematic orbit of the LEO. The code – introduced with its correct weight relative to phase – is required to get the absolute position in space of the phase-connected orbit pieces.

The procedure is very efficient because no ambiguity parameters have to be set up. A limitation of this approach is the fact that correlations between the phase observations are neglected. Additional problems occur at epochs where no position differences can be computed due to not sufficient phase observations (e.g., caused by a loss of phase lock of the receiver). The orbital arcs before and after such epochs are not connected by the phase leading to a jump in the orbit whose magnitude is depending on the accuracy of the code. Finally, the procedure requires an a priori orbit for the LEO which should have an accuracy of a few meters in order not to introduce residual effects into the kinematic orbit solution. The a priori orbit may be generated in a first iteration using code observations only.

The kinematic positions of the satellite may be used as pseudo-observations for a dynamic orbit determination procedure. For experiments the EGM96 or GRIM5 geopotential model to degree and order 95 is used. The procedure allows to model air drag, solar radiation pressure, and albedo. For these three forces a scaling factor may be estimated. In addition the nine parameters of an empirical force model may be determined and stochastic pulses may be inserted at selected epochs. Data from an on-board accelerometer may be introduced in place of modeling non-gravitational forces. To cope with jumps between arcs which are not connected by phase position differences it is possible to estimate offsets between individual orbital arcs. Finally it is also possible to use both, code derived positions and phase derived position differences directly as pseudo observations with correct relative weight for the dynamic orbit determination. In this case, however, no precise kinematic orbit is generated.

Figure 1 shows the differences between a kinematic and a dynamical orbit of SAC-C (day of year 051, 2001) in inertial directions $x, y$ and $z$ after fitting a dynamical orbit through the kinematic positions obtained from the combination of code derived positions and phase derived position differences. The RMS of this dynamical fit is 0.26 m which is mainly due to imperfect dynamic modeling of the orbit. The length of the arc is five revolutions of SAC-C of about 98.5 minutes. The kinematic orbit is connected through position differences over the entire time interval displayed. Nevertheless jumps in the kinematic orbit may be observed. Some of them are indicated by arrows in Figure 1. These jumps are due to bad phase observations affecting the position differences. Elaborate screening algorithms may reduce the number and size of such jumps.

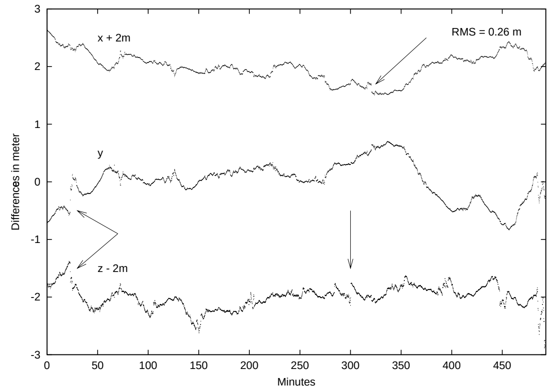


**Fig. 1** Differences in $x, y, z$ between kinematic and dynamic orbit for SAC-C on 01/051. Arrows indicate jumps in the kinematic solution.

## 3. Data Screening

### 3.1 Preprocessing Procedure

Efficient preprocessing and data screening is an important issue for kinematic orbit determination. In the following section we explain our screening procedure in detail as well as the options to modify the performance of the algorithm.

In a first step the code observations are processed for each epoch and the LEO clock is synchronized to GPS time. In the second step the phase differences between subsequent epochs are processed. Both processing steps are preceded by screening procedures.

For simplicity let us have a look at the code observations of the spaceborne GPS receiver for a particular epoch. Usually there are pseudorange observations of up to eight GPS satellites. The code observation equation reads

$$p^j = \rho^j - c \cdot \Delta t^j + c \cdot \Delta t_{LEO} \qquad (1)$$

with the ionosphere free linear combination $p^j$ of the P1- and P2-code measurements to GPS satellite $j$, the geometrical distance $\rho^j$ between GPS satellite $j$ and the LEO, the GPS satellite clock correction $c \cdot \Delta t^j$, and the LEO clock correction $c \cdot \Delta t_{LEO}$.

Using precise GPS orbits and clocks as well as an a priori orbit of the LEO, only the LEO clock correction $c \cdot \Delta t_{LEO}$ remains as unknown in Eq. (1). The fact that the LEO clock correction should be the same for all code observations of one epoch within the accuracy of the code may be used for the data screening. From the statistical point of view this means that the difference between two clock corrections derived from the observation to satellites $i$ and $j$, respectively, should be within $3 \cdot \sigma_{Code}$.

In a first step the difference between each pair of LEO clock corrections is computed and checked if it is smaller than three times a specified $\sigma_{Code}$. Each clock correction thus may be associated with a group of similar values. From the values belonging to the largest of these groups a mean value and RMS are computed. In a second step each clock correction is compared with this mean value. If the difference is larger than a fixed multiple of the computed RMS (e.g., 30 times) the observation is flagged as an outlier and not used in the following point positioning procedure.

After getting rid of the large outliers the point positioning procedure is performed iteratively and additional bad observations may be rejected. The prescreening is nevertheless necessary to remove the large outliers which degrade the point positioning.

The data quality after the preprocessing depends on the screening options. The performance of the algorithm may be changed by modifying the following input parameters:

- the $\sigma_{Code}$ for classifying the observations into groups,
- the RMS for setting the rejection threshold (it may either be derived from the observations or specified as fixed value),
- the factor multiplied with the RMS in order to get the rejection threshold, e.g. 30.

The same pre-screening algorithm is applied to the phase difference observations with the $\sigma_{Code}$ replaced by a $\sigma_{Phase}$. Figure 2 illustrates the procedure. After the pre-screening position differences are generated using only non-flagged observations. If not enough observations are available no solution can be computed for this epoch difference. If the computation of a solution is possible but the corresponding RMS is larger than a specified threshold a series of solutions is computed with one observation removed in turn. The solution with the lowest RMS is used as the final solution. If this RMS still exceeds the threshold the procedure is iterated.

One factor limiting the performance of the preprocessing approach is the accuracy of the a priori orbit. For the point positioning with the code, this is not an issue, but for the screening of the phase differences the quality of the a priori orbit is critical. A bad a priori orbit may mimic bad phase observations which may then be erroneously removed as outliers by the screening algorithm.

Usually we generate an initial a priori orbit using code observations only. No pre-screening is possible in this step. Through the code point positions we

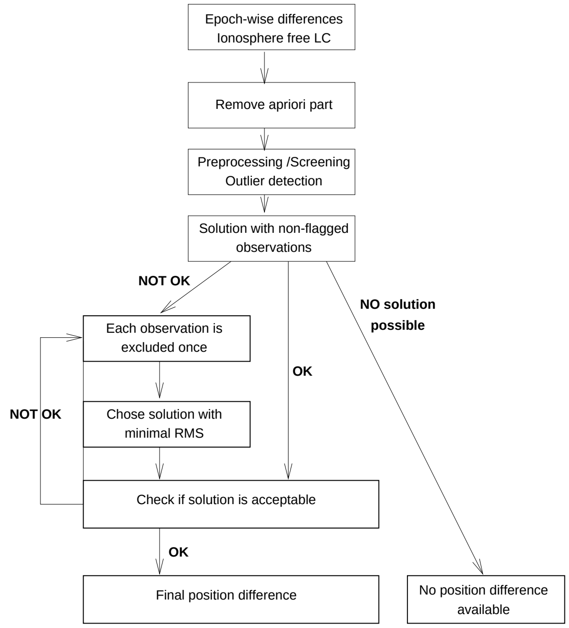


**Fig. 2** Processing scheme for deriving position differences from the phase

fit a dynamical orbit which has an accuracy in the range of a few meters. In a second run this orbit is used to process once again only code observations but with pre-screening enabled. From the kinematic positions from this step an improved dynamic orbit is generated, which is then used for processing both code and phase observations with pre-screening enabled. Finally the kinematic orbit itself may be used as a priori orbit for the generation of an improved kinematic orbit.

## 3.2 Tests and Results

Tests of the preprocessing procedure are carried out with data from CHAMP and SAC-C. CHAMP is designed for gravity field recovery and studies of the magnetic field. It was launched on July 15, 2000 and is orbiting at an altitude of about 430 km in an almost circular and near polar orbit (inclination 87 degrees). SAC-C is an Argentine Earth observation satellite launched on November 23, 2000. The satellite is flying at an altitude of 702 km in a Sun synchronous orbit at an inclination of 98.2 degrees.

Both satellites carry a TurboRogue GPS receiver supplied by JPL/NASA which tracks up to eight satellites at the same time. Occasional data gaps may be attributed to receiver resets or to the downlink periods. For our tests we use data from SAC-C for a particular day having no data gaps (February 20, 2001, DOY 051) and from CHAMP for one day with a single data gap of about 15 minutes (June 1, 2001, DOY 152). The few gaps make the two days ideal for looking into the different options for the preprocessing and data screening algorithm presented in Section 3.1.

Before interpreting results we have to define criteria to select the optimal set of screening options. First of all the number of not connected phase position differences is an important quality indicator for the selection of the preprocessing options. A second quality indicator is the number of jumps in the kinematic orbit exceeding a specified threshold introduced by position differences corrupted by bad observations. Such jumps may be identified by comparing the kinematic orbit with a dynamic orbit.

As a reference we take one solution for each satellite processed with the same options. The relevant options are the following:

1) The RMS value for screening is derived from the observations (it is normally distributed around 5 mm),
2) the threshold for detecting outliers is $30$·RMS,
3) no cut-off angle for the observations is used,
4) elevation-dependent weighting of the observations with the function $w(z) = \cos^2\left(\frac{3}{4} \cdot z\right)$.

In order to get an idea about the influence of the different options on the kinematic solution, different solutions with one or two options changed were computed. Tables 1 and 3 list the different solutions for SAC-C, Tables 2 and 4 for CHAMP. The tables summarize the number of epochs with missing position differences (column 'no connection') as well as the number of jumps larger than 10 cm due to incorrect position differences (column 'Jumps').

For Tables 1 and 2 the outlier rejection threshold for the pre-screening is changed (option 2 in the above list) from $5 \cdot RMS$ to infinity (no outlier rejection). Evidently a screening is necessary (see solution **A6**), otherwise the number of not connected arcs as well as jumps is not acceptable. Not surprisingly the number of jumps in the kinematic orbit due to bad observations is increasing with increasing threshold. On the other hand, a too small threshold leads to an increased number of missing position differences. In solutions **A1** and **A2** for SAC-C probably some good observations are excluded causing problems to connect the positions by the phase at one epoch. For CHAMP the same situation is observed for solution **A1**. It is, in fact, more important to reduce the number of missing position differences to the minimum possible value. Bad observations leading to jumps

**Table 1.** Number of jumps for different kinematic solutions for varying pre-screening threshold - SAC-C

| Solution | RMS× | no connection | Jumps |
|---|---|---|---|
| Ref | 30 | 0 | 34 |
| A1 | 5 | 1 | 5 |
| A2 | 10 | 1 | 9 |
| A3 | 20 | 0 | 22 |
| A4 | 40 | 0 | 44 |
| A5 | 50 | 0 | 47 |
| A6 | ∞ | 8 | 64 |

**Table 2.** Number of jumps for different kinematic solutions for varying pre-screening threshold - CHAMP

| Solution | RMS× | no connection | Jumps |
|---|---|---|---|
| Ref | 30 | 8 | 71 |
| A1 | 5 | 10 | 32 |
| A2 | 10 | 7 | 37 |
| A3 | 20 | 7 | 54 |
| A4 | 40 | 8 | 82 |
| A5 | 50 | 8 | 87 |
| A6 | ∞ | 19 | 115 |

may be excluded in additional pre-screening iterations with variable threshold. A missing position difference may, however, not be recovered.

We have also the possibility to change option 1. We may use a fixed RMS value, e.g., 5 mm for the phase observation screening instead of deriving it from the observations. But no remarkable influence on the results is found.

Recapitulating we can note that the reference solution with the options listed above seems to be optimal for both satellites, CHAMP and SAC-C. The solutions **A3** with a threshold of 20·RMS are somewhat better but one has to keep in mind that the a priori orbit usually has an accuracy of a few meters only which may introduce residual effects by estimating the position differences from phase differences. To avoid this it is safer to take a larger threshold (30·RMS) for the screening algorithm in order not to remove observations erroneously as outliers.

## 4. Elevation-dependent Weighting and Cut-off Angle

In addition to the data pre-screening the cut-off angle as well as the mapping function for the elevation-dependent weighting significantly influence the quality of the resulting kinematic positions. For processing data of ground stations an elevation-dependent weighting is usually applied by the weighting function (Hugentobler *et al.*, 2001)

$$w(z) = \cos^2(z), \tag{2}$$

where $z$ is the zenith angle of the satellite. For observations of a LEO the question arises whether this weighting function is appropriate. For a LEO observations at low elevations are not corrupted by tropospheric refraction but multipath effects may be an important source for degradation of low-elevation data quality, in particular because a LEO may track satellites at zenith angles well above 90 degrees (up to $105 - 110$ degrees). The question is whether these measurements contribute to a better point positioning when applying an appropriate weighting or whether they rather disturb the solution. A simple modification of the weighting function in Eq. (2) is the introduction of a 'stretching factor' $\alpha$, i.e.,

$$w(z) = \cos^2(\alpha \cdot z), \tag{3}$$

in order to use observations with $z \geq 90$ degrees.

Tables 3 and 4 summarizes the results for changing elevation cut-off angle and weighting function (options 3 and 4). The 'stretching factor' $\alpha$ in the weighting function is varied, 'no' in the corresponding column indicates that no elevation dependent weighting is applied. For SAC-C the solution with all observations used and $\alpha = \frac{3}{4}$ seems to be optimal while for CHAMP it seems to be better to have a cut-off angle of 90 degrees and use the weighting function $w(z) = \cos^2(1 \cdot z)$. The reason might be multipath on the surface of the satellite. From the results given in Tables 3 and 4 it is clear that the zenith cut-off angle should not be smaller than 90 degrees.

**Table 3.** Number of jumps for different kinematic solutions with varying zenith cut-off angle and weighting function - SAC-C

| Solution | $z_{max}$ | $\alpha$ | no connection | Jumps |
|---|---|---|---|---|
| Ref | $\infty$ | 3/4 | 0 | 34 |
| D1 | $\infty$ | no | 0 | 34 |
| D2 | 90 | no | 1 | 28 |
| D3 | 90 | 2/3 | 2 | 28 |
| D4 | 90 | 3/4 | 2 | 29 |
| D5 | 90 | 1 | 4 | 29 |
| D6 | 80 | no | 10 | 29 |
| D7 | 80 | 3/4 | 10 | 31 |

**Table 4.** Number of jumps for different kinematic solutions with varying zenith cut-off angle and weighting function - CHAMP

| Solution | $z_{max}$ | $\alpha$ | no connection | Jumps |
|---|---|---|---|---|
| Ref | $\infty$ | 3/4 | 8 | 71 |
| D1 | $\infty$ | no | 8 | 62 |
| D2 | 90 | no | 8 | 50 |
| D3 | 90 | 2/3 | 8 | 61 |
| D4 | 90 | 3/4 | 8 | 59 |
| D5 | 90 | 1 | 7 | 39 |
| D6 | 80 | no | 19 | 44 |
| D7 | 80 | 3/4 | 19 | 43 |

## 5. Comparison of Orbits

For CHAMP we have the possibility to compare our orbits with the so-called Rapid Science Orbits (RSO) computed by GFZ (GeoForschungsZentrum) in Potsdam, Germany. We compute the differences between the RSO of DOY 152 and the kinematic orbit of solution **D5** in Table 4. These differences in the inertial directions $x, y$ and $z$ are plotted in Figure 3 (for $x$ and $z$ an offset of six meters is added). The vertical lines in the plot indicate the epochs where no connection with a position difference derived from phase differences are available. We note that these offsets may have a magnitude of several meters which corresponds to the code accuracy. Due to position differences which are disturbed by bad observations not recognized by the screening procedure, additional jumps can be found in the differences to the RSO (see arrows in Figure 3). The agreement between the kinematic orbit and the CHAMP-RSO is good with an RMS well below one meter. A significant part of this RMS is due to the few large jumps.

Figure 4 shows the differences in $x, y$ and $z$ between a dynamic orbit fit and the CHAMP-RSO. The dynamic orbit is based on kinematic positions from solution **D5** using the GRIM5 gravity field up to degree and order 95 and with introduction of accelerometer data. The first few minutes of kinematic pseudo-observations were removed by a screening built into the orbit determination procedure. The differences between the two orbits are up to several

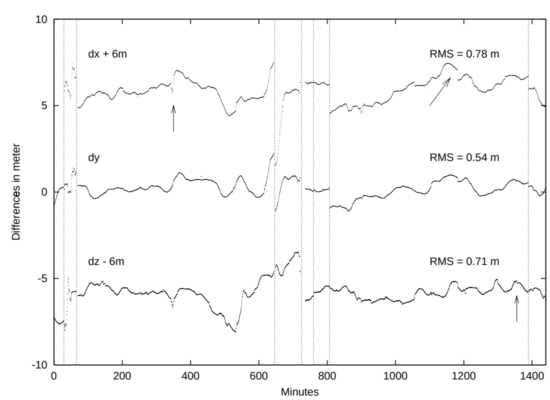


**Fig. 3** Differences between CHAMP - RSO and kinematic orbit **D5** - 01/152.

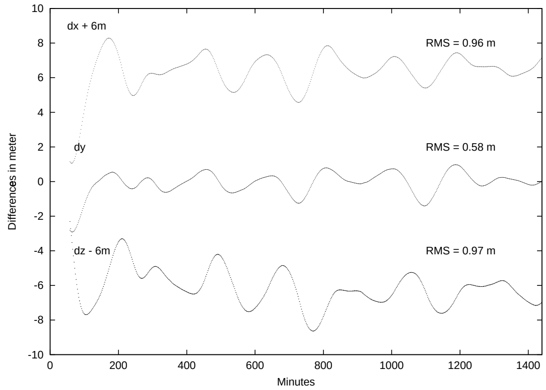


**Fig. 4** Differences between CHAMP - RSO and dynamic orbit accelerometer data used, **D5** is input - 01/152.

meters and mostly due to modeling problems. Nevertheless the RMS difference is below one meter.

## 6. Summary and Outlook

We have developed an algorithm for kinematic point positioning for LEOs based on GPS code and epoch-wise phase difference observations. Experience shows that elaborate screening procedures are required in order to generate a 'clean' kinematic orbit for a LEO. We have developed an algorithm capable of rejecting outliers in a pre-screening step. An a priori orbit which may in a first step be derived from code observations only is a prerequisite. With an optimal set of options the number of orbit disconnections due to missing position differences derived from the phase is minimized. These disconnections lead to jumps in the kinematic orbit of up to several meters.

The elevation cut-off angle should be set to the lowest possible value in order to use a maximum number of observations to strengthen the kinematic solution as long as we have no problems with multipath. For SAC-C the zenith cut-off angle can be lowered well below the local horizon while for CHAMP a cut-off angle of 90 degrees seems to be more appropriate.

We have shown that with an elaborate screening procedure a kinematic orbit with an accuracy at the decimeter level is feasible. On the other hand the dynamic orbit modeling still needs improvement to get satisfactory results.

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