The Art and Science of Precise Orbit Determination

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DLR

Orbit Determination – How it Began

J. Kepler (1609)

*Astronomia Nova seu physica coelestis, tradita commentariis de motibus stellae Martis ex observationibus G.V. Tychonis Brahe**

- Infers trajectory of a body from accurate astronomical observations
- Based on „celestial physics“

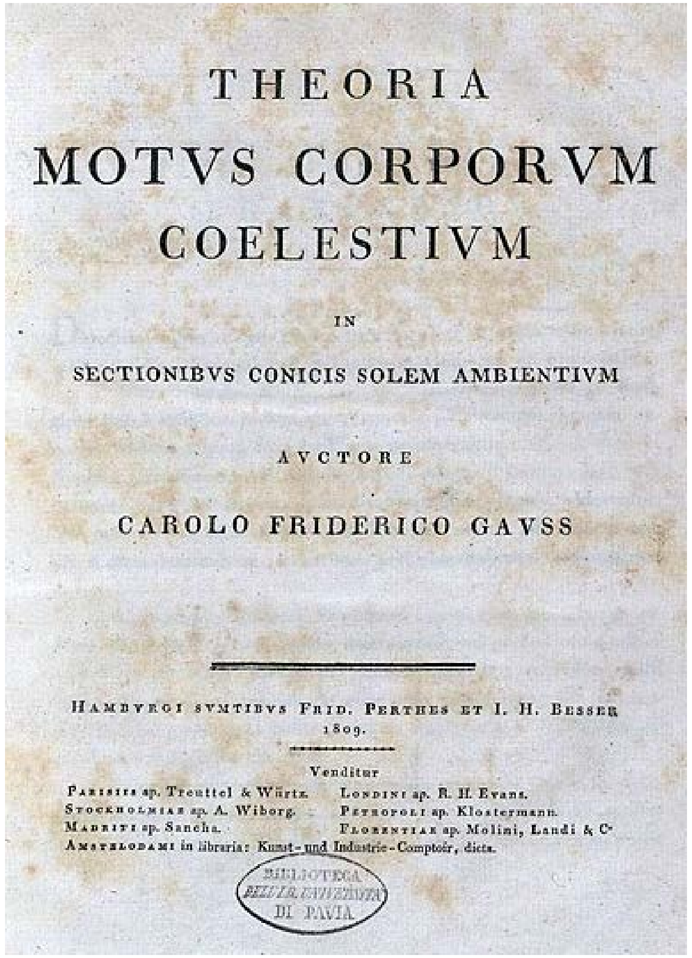
* *New Astronomy, reasoned from Causes, or Celestial Physics, Treated by Means of Commentaries on the Motions of the Star Mars, from the Observations of the noble Tycho Brahe*

Orbit Determination – The Gauss Legacy

Gauss, 1809

- Robust initial orbit determination of newly discovered body (Ceres) from six observations
- Orbit refinement using the method of least-squares
- Consideration of perturbations

C.F. Gauß (1809) Theoria motus corporum coelestium in sectionibus conicis solem ambientium (Theory of the motion of the heavenly bodies moving about the sun in conic sections)



THEORIA
MOTVS CORPORVM
COELESTIVM
IN
SECTIONIBVS CONICIS SOLEM AMBIENTIVM
AVCTORE
CAROLO FRIDERICO GAVSS

HAMBVRGI SVMTIBVS FRID. PERTHES ET I. H. BESSER
1809.

Venditur
PARISIIS ap. Treuttel & Würtz. LONDINI ap. R. H. Evans.
STOCKHOLMIAE ap. A. Wiborg. PETROPOLI ap. Klostermann.
MADRITI ap. Sancha. FLORENTIAE ap. Molini, Landi & C°
AMSTELODAMI in libraria: Kunst- und Industrie-Comptoir, dicta.

Image credits: Wikipedia

Orbit Determination of Artificial Satellites

~50000 catalogued objects

3500 active satellites (early 2021)
- 1800 Communication (GEO & LEO)
- 900 Earth Observation (mostly LEO)
- 150 Navigation (MEO)

Initial orbit determination
- Space debris

Orbit determination
- Mission operations support

Precise orbit determination
- Navigation
- Remote sensing
- Geodesy

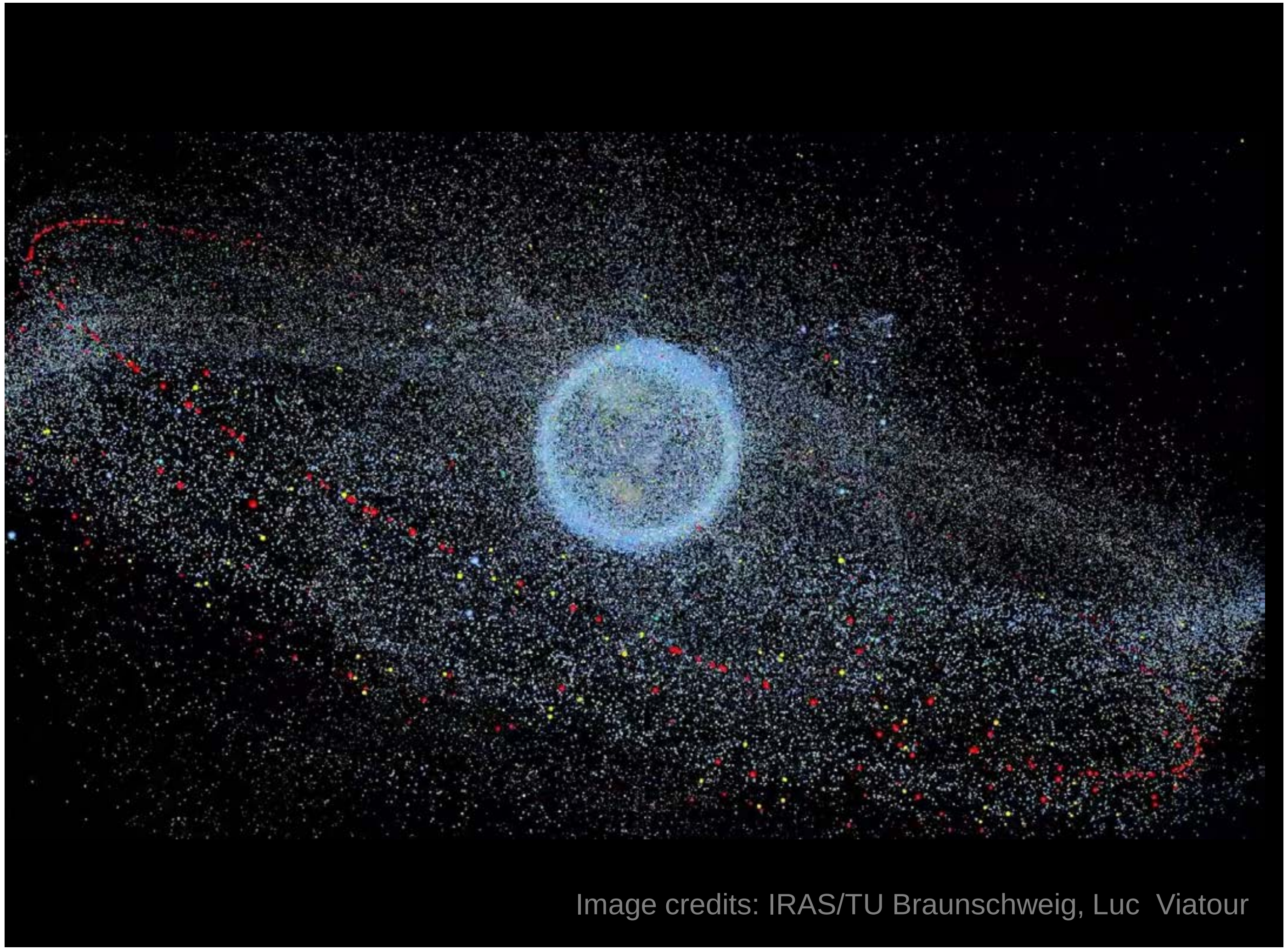
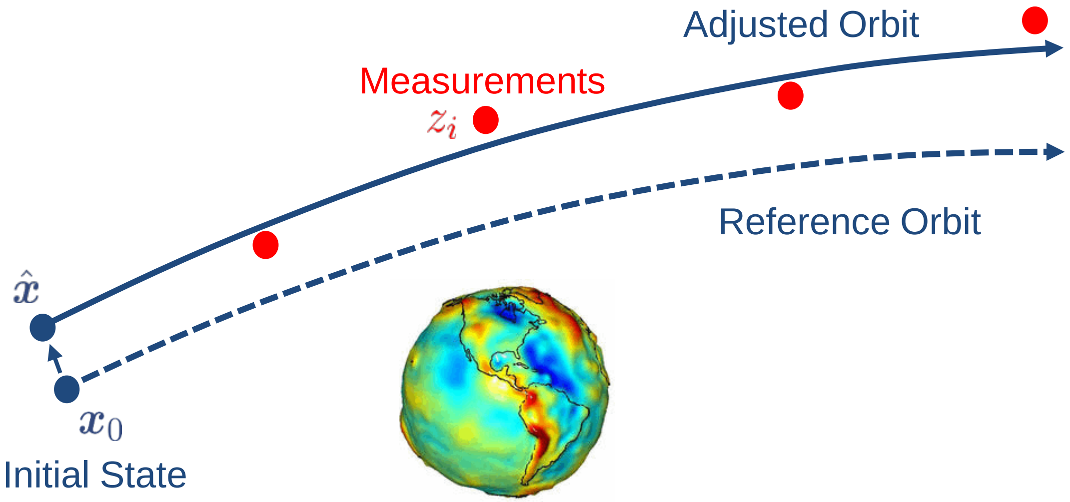


Image credits: IRAS/TU Braunschweig, Luc Viatour

Precise Orbit Determination in a Nutshell

- Dynamical model $\ddot{\mathbf{r}} = \mathbf{a}(t,\mathbf{r},\mathbf{v})$
- Numerical integration $(\mathbf{r},\mathbf{v}) = (\mathbf{r}_0,\mathbf{v}_0) + \int_{t_o}^{t} (\mathbf{v},\mathbf{a}(t',\mathbf{r},\mathbf{v}))dt'$
- Measurement model $g(t,\mathbf{r},\mathbf{v}) = h(t,\mathbf{r}_0,\mathbf{v}_0)$
- Least squares adjustment $J = (\mathbf{z}-\mathbf{h}(\mathbf{r}_0,\mathbf{v}_0))^T\mathbf{W}(\mathbf{z}-\mathbf{h}(\mathbf{r}_0,\mathbf{v}_0))$



AIUB contributions

- Earth gravity field models
- Empirical CODE orbit model (ECOM)
- Non-gravitational force models
- Collocation method
- Antenna phase center modeling/estimation
- Earth orientation, reference frame, site models
- Bernese GNSS Software

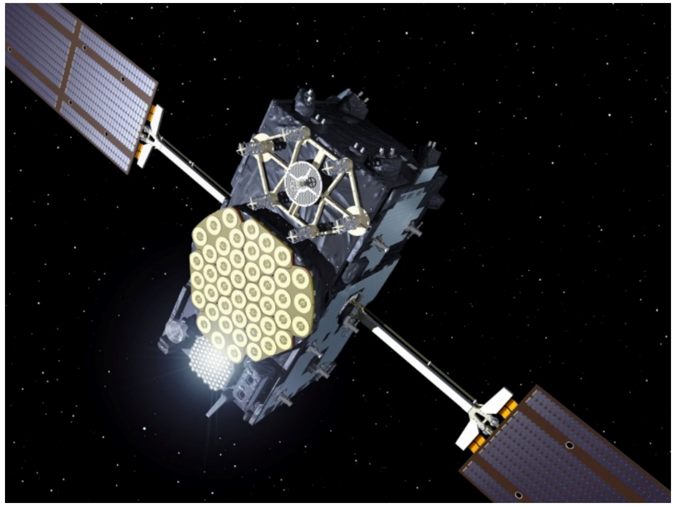
Applications

- GNSS orbits, clocks, geodetic parameters
- Champ, GRACE, GOCE, Metop, COSMIC, Swarm, Sentinel
- Gravity field models

Global Navigation Satellite Systems (GNSS)

- 4 constellations (GPS, GLONASS, Galileo, BeiDou)
- 24-32 satellites in medium Earth orbit (24’-30’ km)
- L-band microwave signals (1.1-1.6 GHz)
  - Pseudo-random noise ranging codes (dm-level noise)
  - Carrier-phase range (ambiguous, mm-level noise)
- “One-way” measurements (signal travel time difference as measured by receiver and onboard clock)
- Highly stable on-board clocks
- Continuous measurements from global monitoring network (International GNSS Service, IGS)

Accurate knowledge of GNSS satellite position and clock offset is a prerequisite for point positioning and geodetic use of GNSS signals



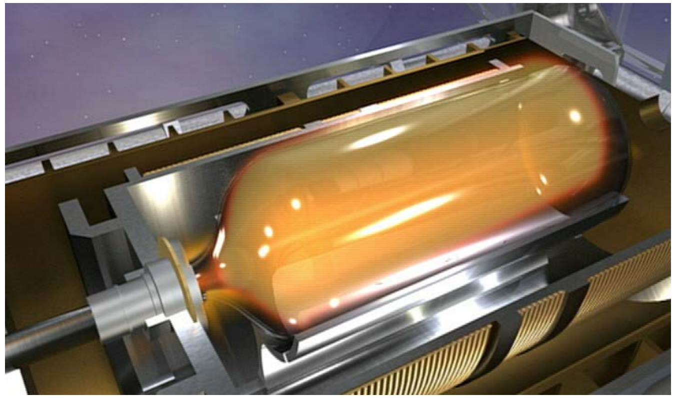
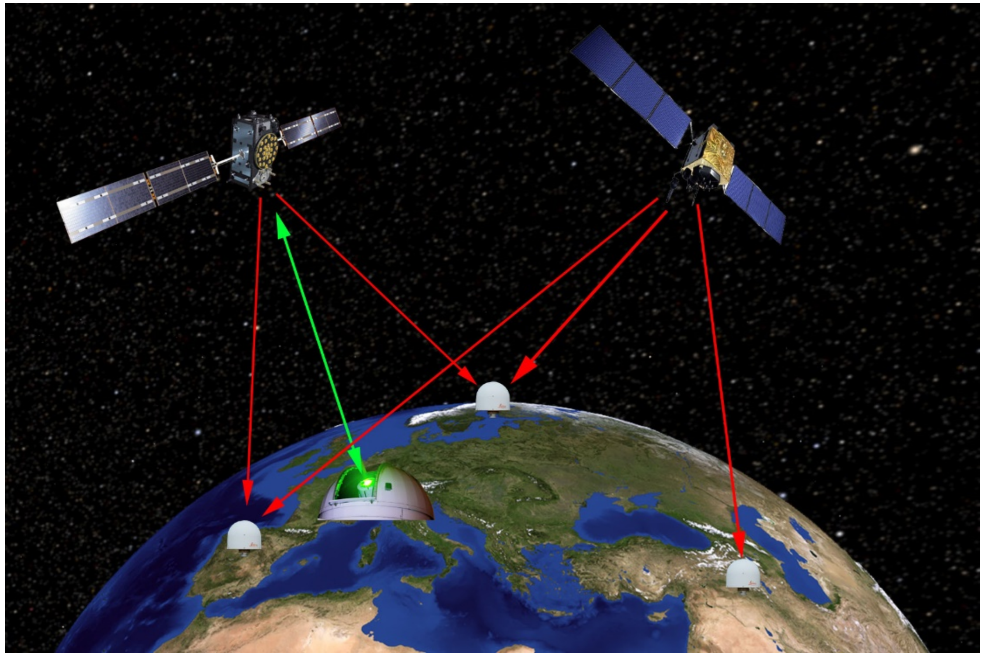
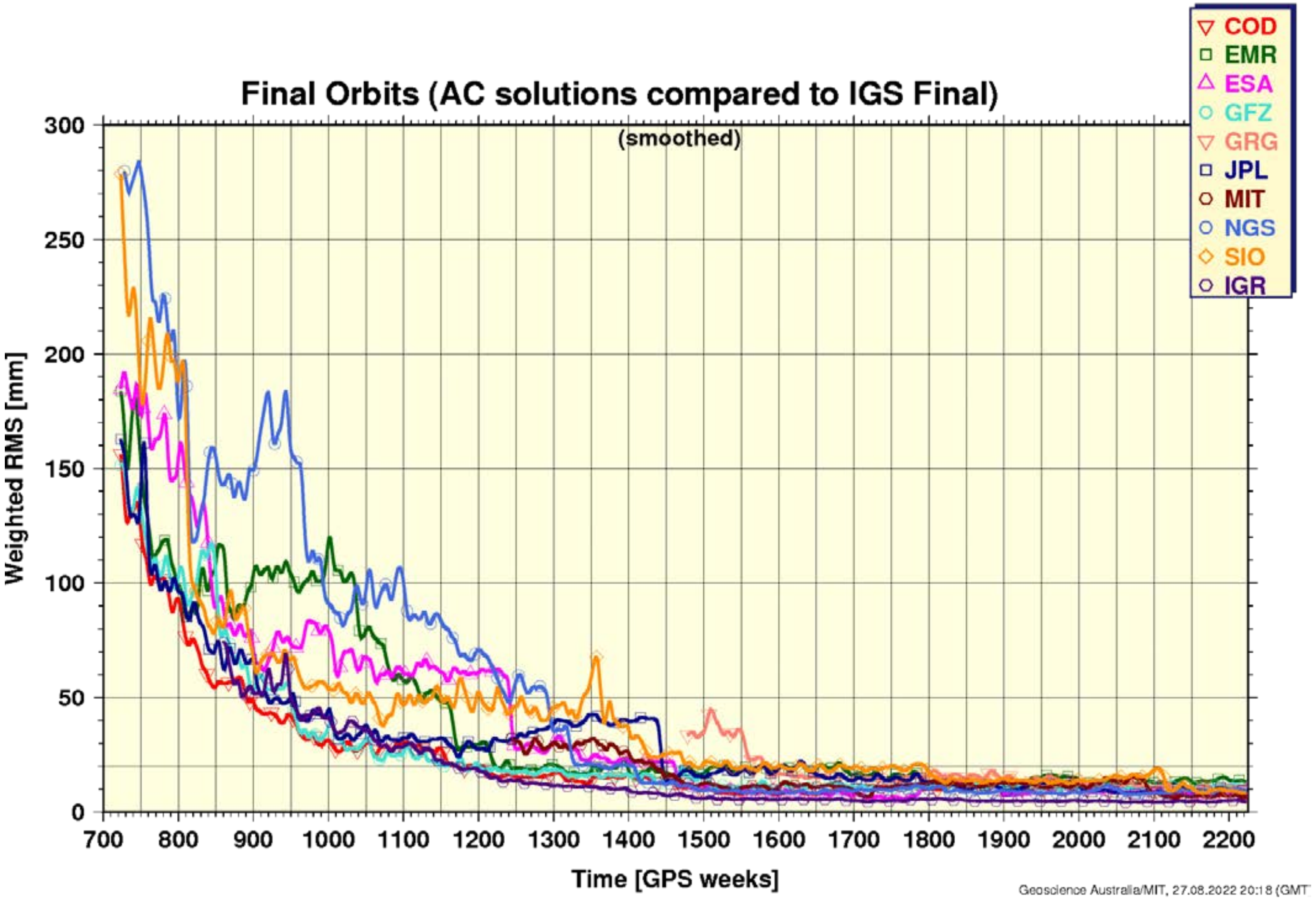


Image credits: ESA

GNSS Orbit Determination and Time Synchronization

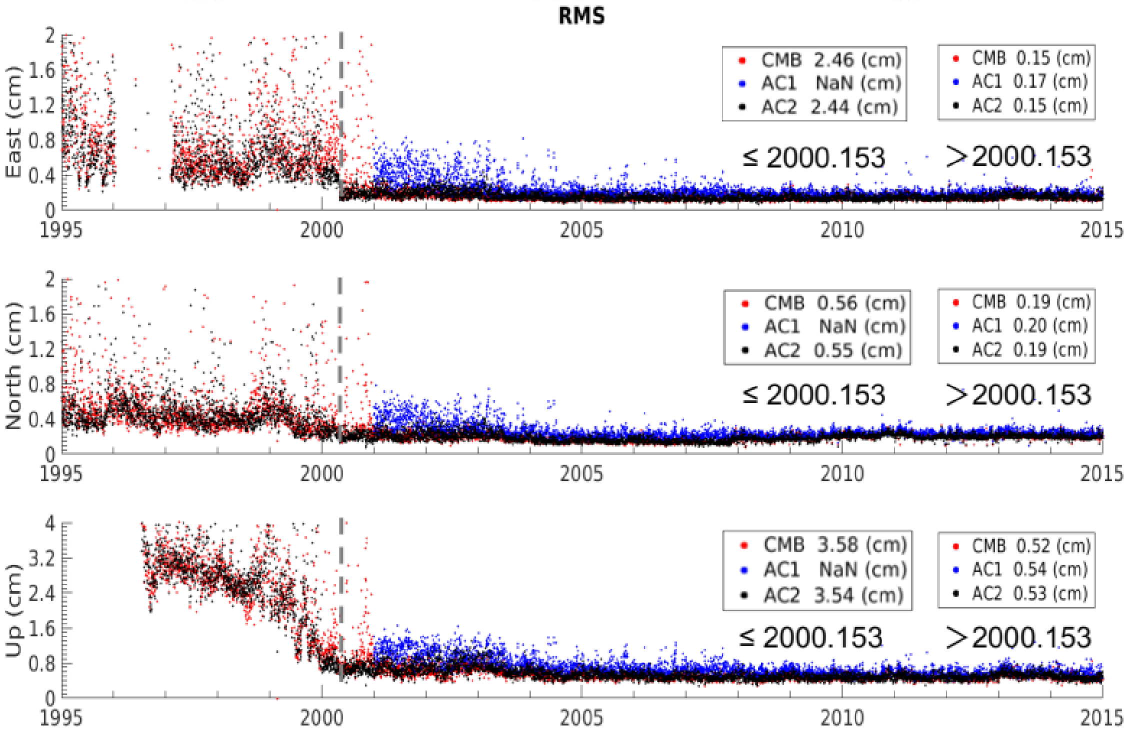




- Routine processing within the IGS
- Consistency among Analysis Centers improved from 2 dm (1992) to 1 cm (2022)

Achievements

- Empirical and box-wing radiation pressure models
- Earth radiation pressure and antenna thrust models
- Time varying gravity models
- Absolute receiver phase pattern calibration
- GNSS transmit antenna pattern calibration
- Sub-daily Earth orientation parameters
- Non-tidal loading effects on station coordinates
- Troposphere and higher-order ionosphere models
- “Reverse kinematic PPP” for eclipse attitude
- Phase bias products for ambiguity fixing
- Combination of orbit & clock products



(J.Geng, IGS W/S 2022)

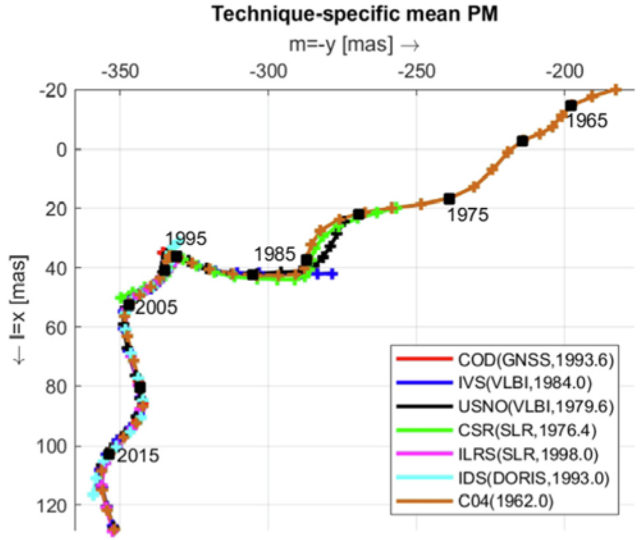
- 3/5 mm horizontal/vertical precise point positioning accuracy

GNSS for Geodesy

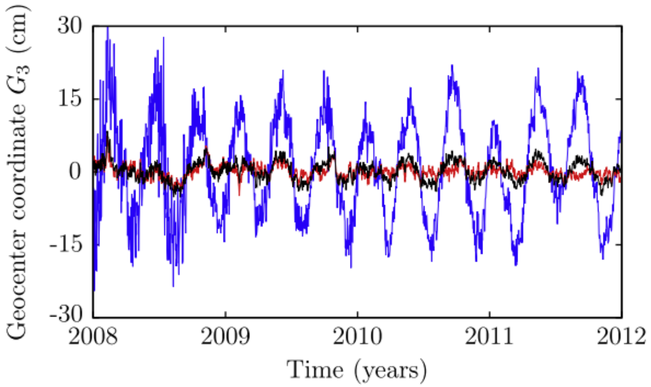
- Estimation of geodetic parameter series as part of POD
    - Station coordinates and velocities
    - Earth rotation
    - Geocenter motion (center-of-mass offset from center-of-figure)
- International Terrestrial Reference Frame (ITRF)
    - Convenient ITRF access (dense reference point network)
    - Independent GNSS-based scale (?)

Problems

- “Draconitic” signals reflecting the period of the Sun’s motion w.r.t. to the GNSS orbital plane (351.6 d for GPS) and harmonics show up in all geodetic GNSS products



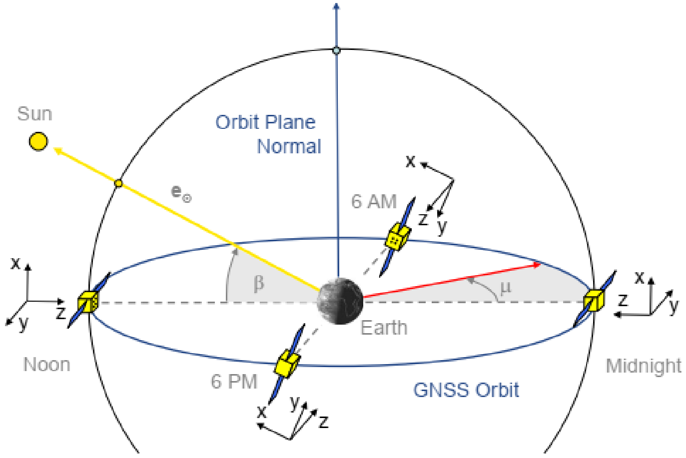
Beutler et al. (2020) Long polar motion series: Facts and insights ASR 66(11):2487-2515. DOI 10.1016/j.asr.2020.08.033

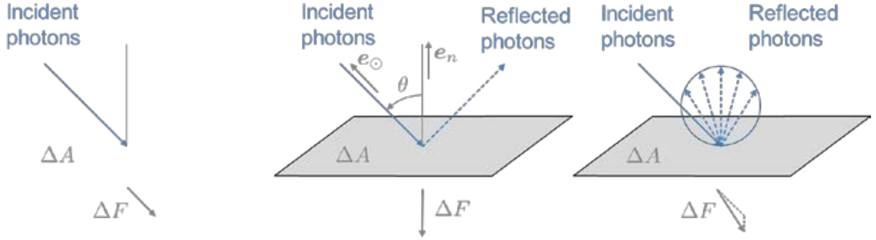


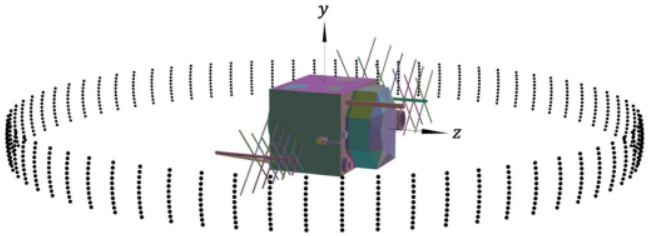
Meindl et al (2013) Geocenter coordinates estimated from GNSS data as viewed by perturbation theory. ASR 51(7):1047-1064.

GNSS Solar Radiation Pressure Modeling

- Dominant non-gravitational force for GNSS satellites
- Varying orientation of satellite body and solar panels
- Empirical models compensate/estimate unmodelled accelerations with “smart” parameterization (e.g. ECOM)
- Physical modeling based on shape and material properties
  - Box-wing
  - Raytracing
- SRP modeling clearly reduces the draconitic signals
- Challenges
  - Eclipses
  - Non-standard attitude at noon-and midnight turns
  - Thermal radiation
  - Unknown satellite properties, diverse platforms



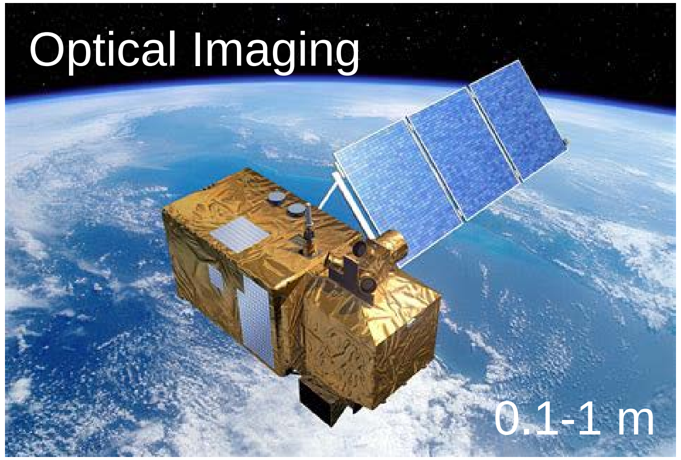


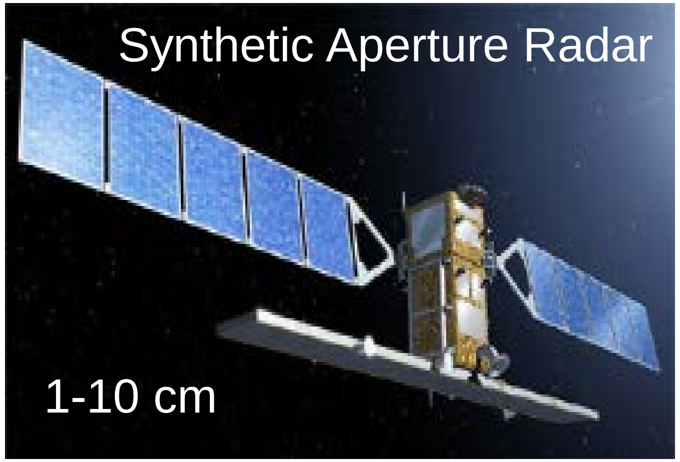


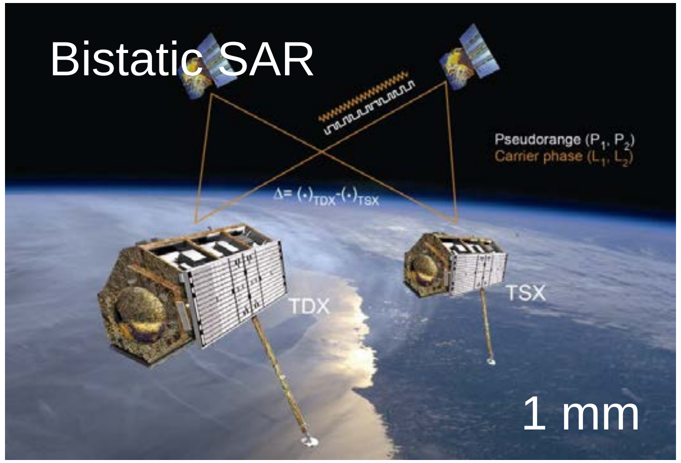
(Source: Bhattarai et al. (2019) JGeod 93:1515–1528)

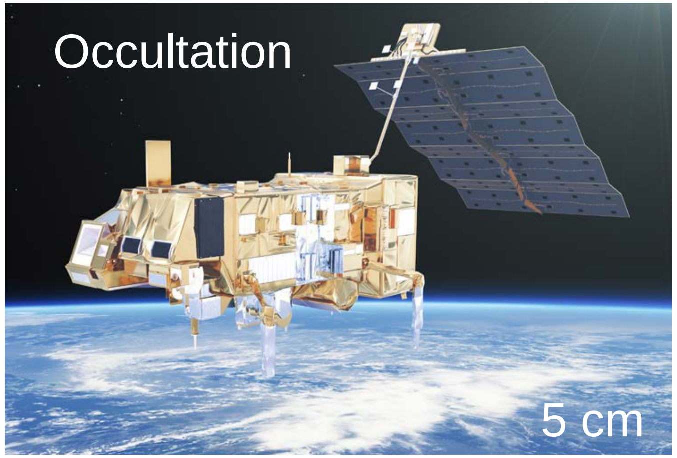
Springer et al (1999) Improving the orbit estimates of GPS satellites. JGeod. 73(3):147-157
Arnold et al. (2015) CODE’s new solar radiation pressure model for GNSS orbit determination. JGeod 89(8):775-791
Sidorov et al. (2020) Adopting the empirical CODE orbit model to Galileo satellites. ASR 66(12):2799-2811.

Precise Orbit Determination of Satellites in Low Earth Orbit (LEO)

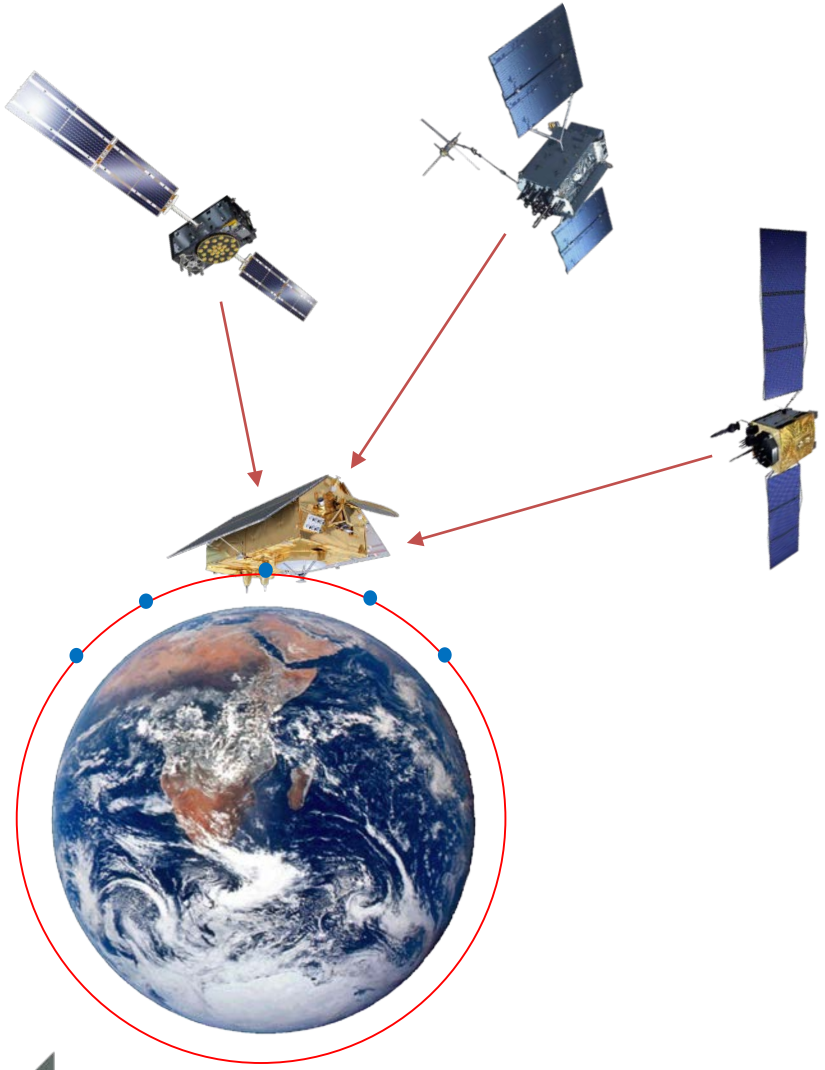




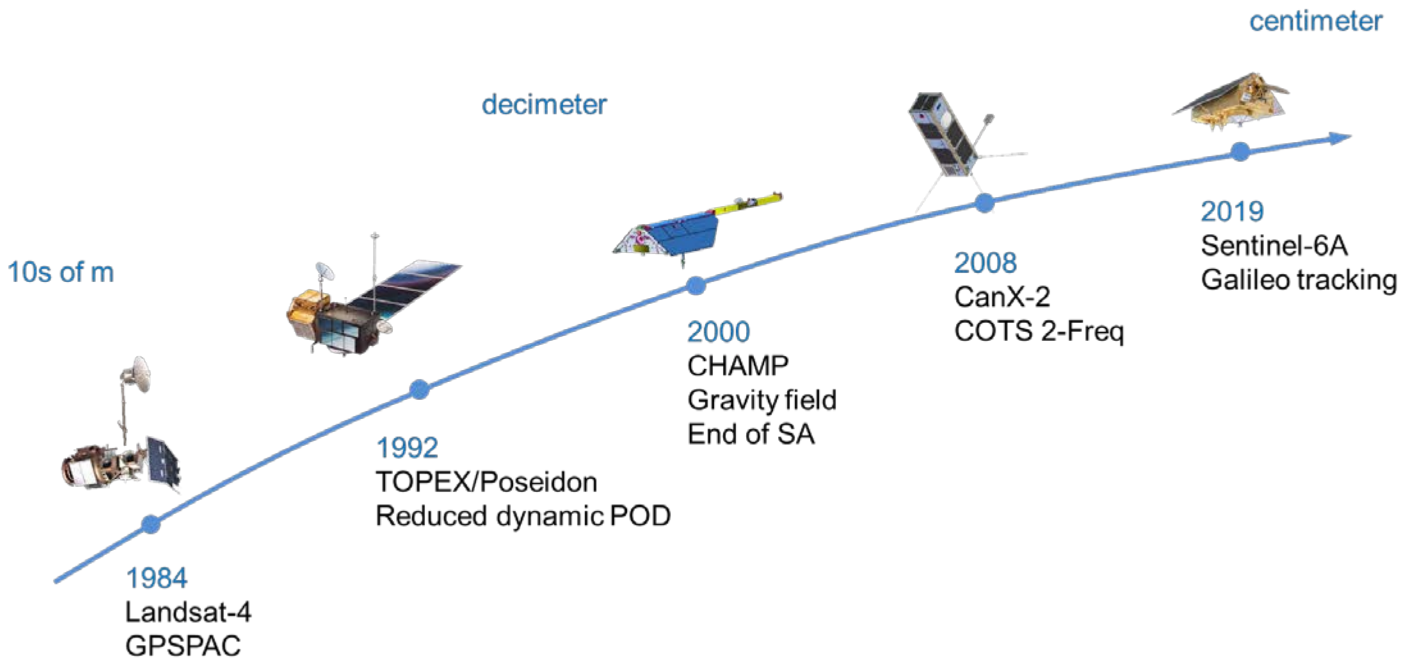




GNSS-Based POD



- High accuracy
  - Millimeter-level precision of GNSS carrier phase
  - Strong geometry (multiple concurrent signal sources)
- Global and, optionally, autonomous
- Dynamic or kinematic POD



centimeter

decimeter

10s of m

1984
Landsat-4
GPSPAC

1992
TOPEX/Poseidon
Reduced dynamic POD

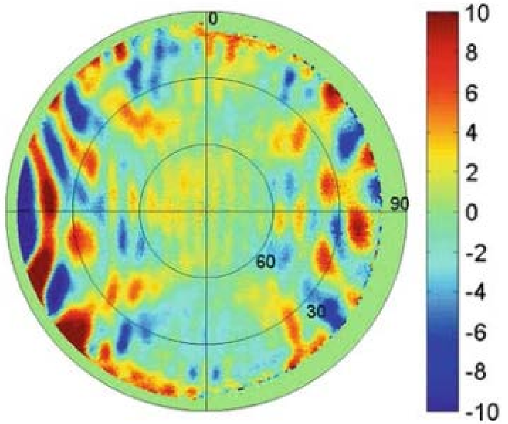
2000
CHAMP
Gravity field
End of SA

2008
CanX-2
COTS 2-Freq

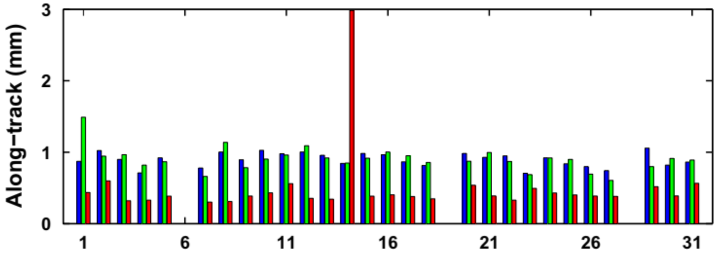
2019
Sentinel-6A
Galileo tracking

Achievements …

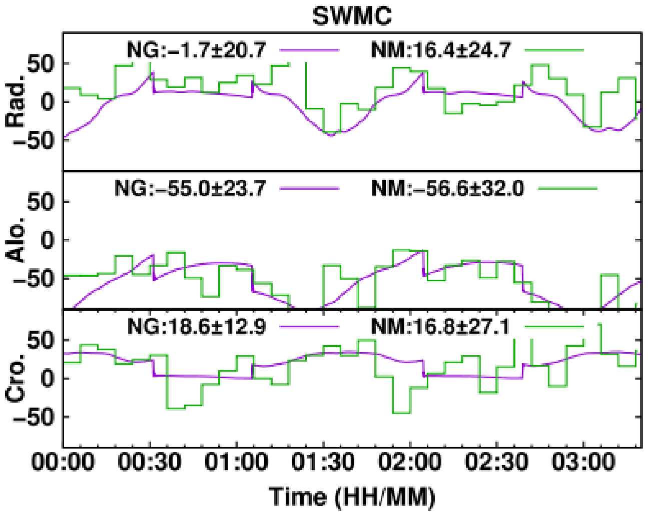
- Phase pattern calibration
- Relative orbit determination (GRACE, TanDEM-X, Swarm)
- Kinematic POD
- Single-receiver ambiguity fixing
- Non-gravitational force modeling
- Time varying gravity fields
- Copernicus POD Service, COSMIC, Eumetsat
- …



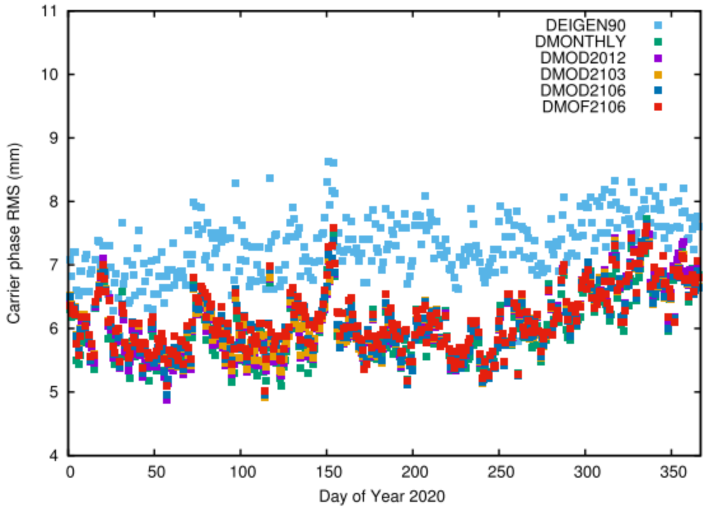
Jäggi et al. (2009) Phase center modeling for LEO GPS receiver antennas and its impact on precise orbit determination. JGeod 83(12):1145



Jäggi A (2012) Inter-agency comparison of TanDEM-X baseline solutions. ASR 50(2):260



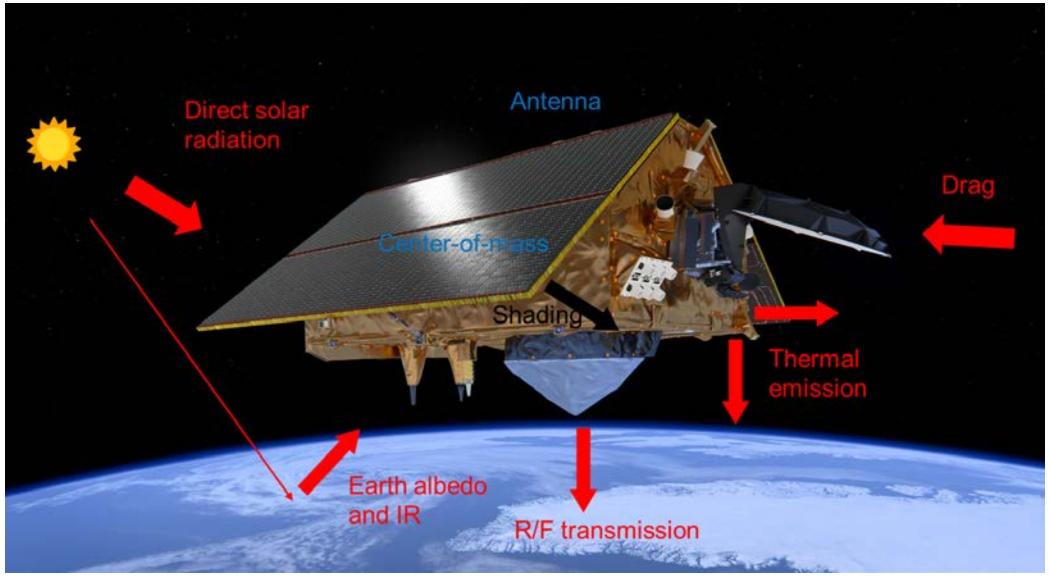
Mao et al. (2021) Dynamic GPS-based LEO orbit determination with 1 cm precision using the Bernese GNSS Software. ASR 67(2):788-805

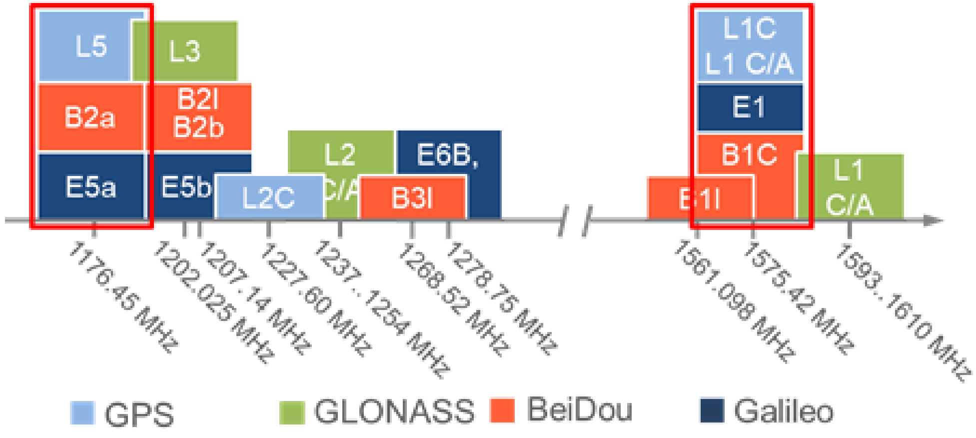


Peter et al. (2022) COST-G gravity field models for precise orbit determination of LEO Satellites. ASR 69(12):4155

… and Challenges

- Non-gravitational force modeling
  - Characterization of spacecraft properties
  - 10 nm/s^2 → 1 cm
- Multi-GNSS
  - Consistent processing of diverse signals and frequency bands
  - New receivers and antennas
- (Near-)real-time POD
  - Few cm accuracy “as soon as possible”
  - Onboard POD
  - Use of GNSS correction services
- LEO (Mega-)Constellations
  - Excessive number of satellites
  - New signals and equipment
  - Near-real time POD?

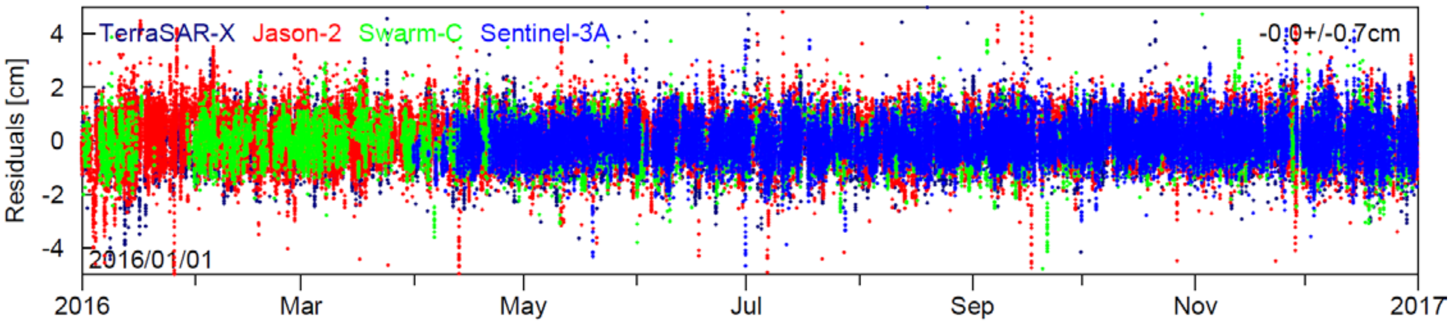




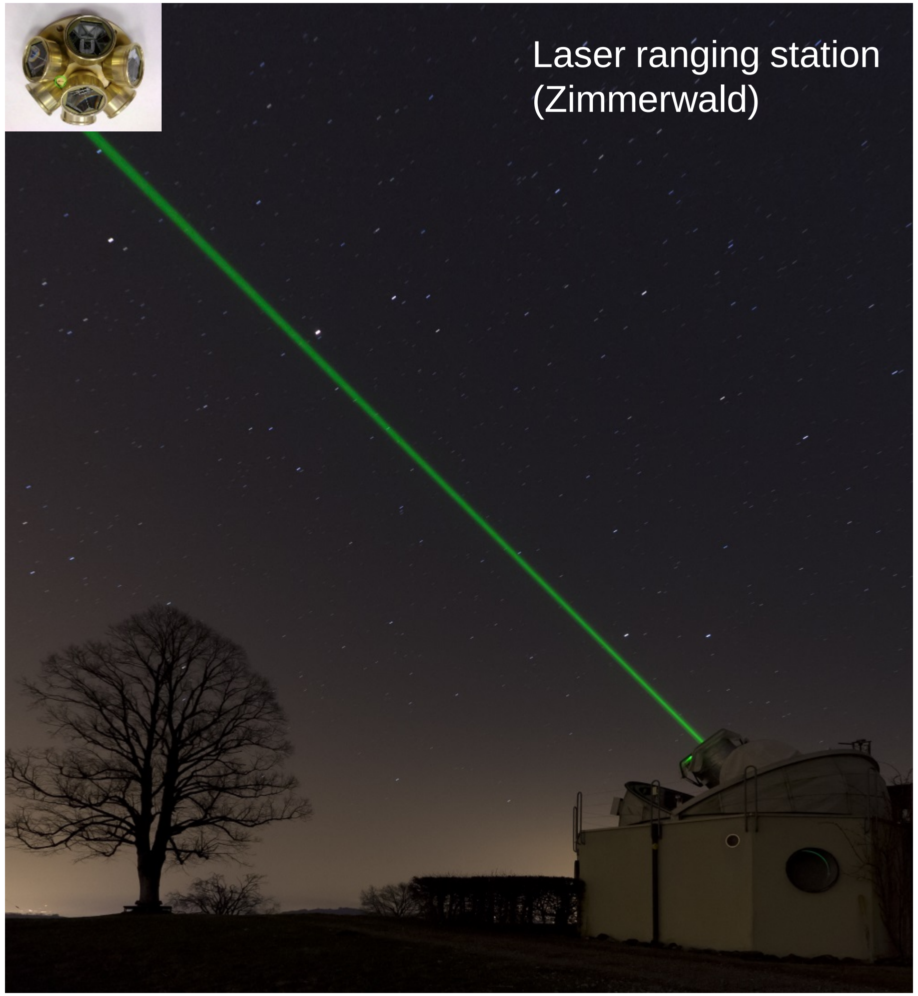
Satellite Laser Ranging (SLR)

How accurate is “precise” orbit determination?

- Two-way light time measurements (mm precision)
- Primary technique for independent LEO orbit validation
- Highly accurate modeling of sites, sensors, atmosphere
- GNSS/SLR frame tie: GNSS-based LEO orbits can contribute to improving SLR reference frame
- 7 mm consistency (range), 1 cm 3D rms accuracy!



Arnold et al. (2019) Satellite Laser Ranging to Low Earth Orbiters: Orbit and Network Validation; JGeod 93(11):2315-2334 (2019) DOI 10.1007/s00190-018-1140-4


Laser ranging station (Zimmerwald)

Summary and Outlook

- POD is a fundamental building block for navigation, remote sensing, and geodesy
- AIUB and the Center for Orbit Determination in Europe have made vital contributions to advancing the start of the art
- Continuous quest for better accuracy and reduced timeliness
  - 1 mm terrestrial reference frame
  - (Sub-)cm LEO POD in (near-)real-time

Challenges

- Permanent model refinement (Earth, satellites, sensors) to minimize the need for estimated parameters
- New measurements systems (inter-satellite links, new signals, optical measurements)
- Mega-constellations in LEO
- High accuracy in real-time